MATH MATH MATH

ALGEBRA I – 8

1104

Name______________________________________

Date Issued	Test Score

SCHOOL OF TOMORROW®

I. REVIEW OF EXPONENTS

Multiplication

OBJECTIVE

To review multiplication of exponential terms

An exponent is a small numeral or variable written at the upper right of a base to show how many times the base is used as a factor.

8^3 means 8 is used as a factor three times. 8 is the base and 3 is the exponent.

$$8^3 = 8 \bullet 8 \bullet 8 = 512$$

To multiply exponential terms which have the same base, add the exponents. The base is unchanged.

$$2^2 \bullet 2^3 = 2^{2+3} = 2^5$$
$$4 \bullet 8 = 32$$

$$x^2 \bullet x^4 = x^{2+4} = x^6$$
$$(x \bullet x)(x \bullet x \bullet x \bullet x) = x \bullet x \bullet x \bullet x \bullet x \bullet x$$

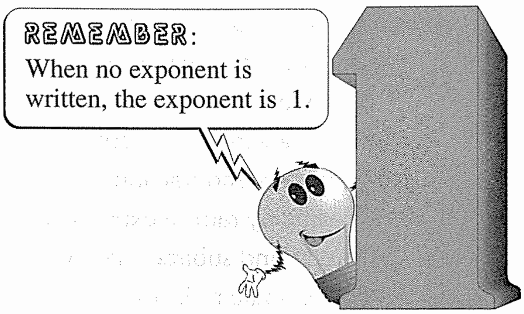
$$(x^{2a+3b})(x^{a-b}) = x^{2a+3b+a-b} = x^{3a+2b}$$

$$3ab^2 \bullet 4a^2b^2 = 12a^{1+2}b^{2+2} = 12a^3b^4$$

$$(a-b)^{2x}(a-b)^x = (a-b)^{2x+x} = (a-b)^{3x}$$

$$a^3 \bullet a^2 \bullet a = a^{3+2+1} = a^6$$

$$x^2 \bullet x^3 \bullet y^2 \bullet y^3 = x^{2+3}y^{2+3} = x^5y^5$$



Write the following products using exponential form.

1. $2^3 \bullet 2^5 =$ ________________

2. $n^3 \bullet n^4 =$ ________________

3. $2 \bullet 2^3 \bullet 2^4 =$ ________________

4. $a^5 \bullet a^2 =$ ________________

5. $9 \bullet 9^8 =$ ________________

6. $x^{3a} \bullet x^{2a} =$ ________________

7. $x^{3a} \bullet x^b =$ ________________

8. $3^2 \bullet 3^4 =$ ________________

9. $b^3 \bullet b =$ ________________

10. $a \bullet a^2 \bullet a^3 =$ ________________

11. $x^{100} \bullet x^{100} =$ ________________

12. $x^9 \bullet x^2 =$ ________________

13. $x^a \bullet x^{2a} \bullet y^2 =$ ________________

14. $(x^{2a+7})(x^{2a-8}) =$ ________________

15. $(x+2)^2(x+2)^2 =$ ________________

16. $(x+y)^{a+b}(x+y)^{a-b} =$ ________________

☐ Score this page. ☐ Correct mistakes. ☐ Rescore.

Exponents and Parentheses

OBJECTIVE

To review the rules that apply to exponents and parentheses

In an algebraic expression, the exponent refers only to the number or variable to its immediate left. In "xy^2," the exponent 2 applies only to the y, not to the x.

$$xy^2 = x \bullet y \bullet y$$

In "$3x^2$," the exponent applies to the x, but not to the 3.

$$3x^2 = 3 \bullet x \bullet x$$

However, when an exponent is written outside a parenthetical expression, the exponent applies to everything inside the parentheses.

$$(xy)^2 = (xy)(xy) = x^2y^2 \qquad (3x)^2 = (3x)(3x) = 9x^2$$

Study the following examples. Notice the differences in the signs of the products.

$$(-2x)^4 = (-2x)(-2x)(-2x)(-2x) = 16x^4$$

$$-(2x)^4 = -(2x)(2x)(2x)(2x) = -16x^4$$

$$-3^2 = -(3)(3) = -9$$

$$(-3)^2 = (-3)(-3) = 9$$

Remove parentheses where necessary and simplify.

1. $(-x)^6 =$ ________________
2. $3(xy)^3 =$ ________________
3. $(4xy)^2 =$ ________________
4. $-x^4 =$ ________________
5. $-2(xy)^2 =$ ________________
6. $(-2x)^3 =$ ________________
7. $(3xy)^3 =$ ________________
8. $(-x)^4 =$ ________________
9. $(-2xy)^2 =$ ________________
10. $5(-xy)^3 =$ ________________

For an exponent to refer to both the numerator and denominator of a fraction, the fraction must be written within parentheses, with the exponent outside the parentheses.

$$\left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9} \qquad \frac{2^2}{3} = \frac{(2)(2)}{3} = \frac{4}{3} = 1\frac{1}{3} \qquad \frac{2}{3^2} = \frac{2}{(3)(3)} = \frac{2}{9}$$

Do you see the difference the parentheses made in the first example?

Fractional Exponents

OBJECTIVE

To understand fractional exponents

There are two ways to indicate that a root of a number is desired. We are already familiar with the most common—the radical sign. Another way to indicate a root is by using fractional exponents. The fractional exponent, $\frac{1}{2}$, means "the square root of"; the fractional exponent, $\frac{1}{3}$, means "the cube root of"; etc.

$25^{\frac{1}{2}} = \sqrt{25} = 5$ $\qquad$ $81^{\frac{1}{4}} = \sqrt[4]{81} = 3$

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$ $\qquad$ $-(49)^{\frac{1}{2}} = -\sqrt{49} = -7$

$(-64)^{\frac{1}{3}} = \sqrt[3]{-64} = -4$ $\qquad$ $(27a^6b^9)^{\frac{1}{3}} = \sqrt[3]{27a^6b^9} = 3a^2b^3$

Find the indicated roots.

1. $-(100)^{\frac{1}{2}} =$

2. $(27x^6y^3)^{\frac{1}{3}} =$

3. $\left(\frac{4x^4}{81y^2}\right)^{\frac{1}{2}} =$

4. $(-125c^3y^9)^{\frac{1}{3}} =$

5. $(-8)^{\frac{1}{3}} =$

6. $(-32x^5y^{10})^{\frac{1}{5}} =$

7. $(81a^4x^{12})^{\frac{1}{4}} =$

8. $16^{\frac{1}{4}} =$

We must be courageous, but also reasonable. The world admires us for walking a tightrope without falling off. It asks us to keep our balance.

Lech Walesa
(Former president of Poland)

☐ Score this page. ☐ Correct mistakes. ☐ Rescore.

CHECKUP
(4 points each question)

My score ______

Simplify these expressions.

1. $-(4x^2y^2)^4 =$

2. $(-3^2)^2 =$

3. $(3x^2y^2)^4 =$

4. $\dfrac{x^2(x^3)^2}{x^{10}} =$

5. $\left(\dfrac{-3a^2}{4b^2}\right)^3 =$

6. $\dfrac{(5x^2y)^2(-100y^3)}{(25y)^2} =$

7. $\dfrac{(3ab^2)(-6a^3b)}{(3a^2b^2)^2} =$

8. $4(x^2)^0 =$

9. $(-2ab)^{-3} =$

10. $4y^2 \bullet 4x^2 \bullet 3(xy)^0 =$

11. $x^{-4} =$

12. $\left(\dfrac{3}{4}\right)^{-2} =$

13. $\left(\dfrac{x}{2}\right)^{-3}\left(\dfrac{8y^2}{x^2}\right)^0 =$

14. $\dfrac{(-3a^2b^2c^2)^0}{3ab} =$

15. $(4x^2)^{-3} =$

16. $(-x)^{-2} =$

Find the indicated roots.

17. $\sqrt{25x^2y^4} =$

18. $\sqrt{\dfrac{1}{49}} =$

19. $\left(\dfrac{1}{1{,}000x^6}\right)^{\frac{1}{3}} =$

20. $\sqrt[4]{16a^8b^4} =$

21. $(36x^2y^4z^2)^{\frac{1}{2}} =$

22. $\sqrt[3]{-27x^3y^6} =$

Complete these statements.

23. The quantity under the radical sign is called the ______________________.

24. The positive square root is called the ____ .

25. Fractional exponents indicate a/an ______________________ of a number is desired.

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10. $3\sqrt{99} - 2\sqrt{44} - 6\sqrt{11} = ?$

11. $\sqrt[3]{16} + 2\sqrt[3]{128} - \sqrt[3]{54} = ?$

12. $3\sqrt[4]{48} - 2\sqrt[4]{243} = ?$

☐ **Score pages 20, 21, and 22.** ☐ **Correct mistakes.** ☐ **Rescore.**

Rationalizing the Denominator

OBJECTIVE

To eliminate radicals from the denominators of fractional terms

If a fractional term contains radicals in its denominator, it is customary to eliminate them by multiplying the fraction by a suitable expression of 1. Such elimination is called ***rationalizing the denominator***. The numerator and denominator of a fraction may be multiplied by the same number without changing the value of the fraction. To rationalize the denominator, choose the smallest multiple of the radicand which is a perfect square, cube, fourth power, or whatever power is indicated by the index.

Rationalize the denominator of $\sqrt{\frac{9}{8}}$.

1. Simplify the numerator, if possible.

$$\sqrt{\frac{9}{8}} = \frac{\sqrt{9}}{\sqrt{8}} = \frac{3}{\sqrt{8}}$$

2. Choose the smallest multiple of the radicand which is a perfect square.
   The smallest multiple of 8 which is a perfect square is 16. Therefore, we must multiply both numerator and denominator by $\sqrt{2}$.

3. Multiply.

$$\frac{3\sqrt{2}}{\sqrt{8}\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{16}}$$

4. Simplify the denominator. Reduce if necessary.

$$\frac{3\sqrt{2}}{\sqrt{16}} = \frac{3\sqrt{2}}{4}$$

STUDY THE FOLLOWING EXAMPLES CAREFULLY.

$$\sqrt{\frac{9}{15}} = \frac{\sqrt{9}}{\sqrt{15}} = \frac{3}{\sqrt{15}} = \frac{3\sqrt{15}}{\sqrt{15}\sqrt{15}} = \frac{3\sqrt{15}}{\sqrt{225}} = \frac{\overset{1}{\cancel{3}}\sqrt{15}}{\underset{5}{\cancel{15}}} = \frac{\sqrt{15}}{5}$$

(The smallest multiple of 15 which is a perfect square is 225.)

$$\sqrt[4]{\frac{1}{4}} = \frac{\sqrt[4]{1}}{\sqrt[4]{4}} = \frac{1}{\sqrt[4]{4}} = \frac{1\sqrt[4]{4}}{\sqrt[4]{4}\sqrt[4]{4}} = \frac{\sqrt[4]{4}}{\sqrt[4]{16}} = \frac{\sqrt[4]{4}}{2}$$

(The smallest multiple of 4 which is a perfect fourth power is 16.)

$$\frac{2x^2}{3\sqrt{2x}} = \frac{2x^2\sqrt{2x}}{3\sqrt{2x}\sqrt{2x}} = \frac{2x^2\sqrt{2x}}{3\sqrt{4x^2}} = \frac{\overset{1}{\cancel{2}}\overset{x}{\cancel{x^2}}\sqrt{2x}}{3(\underset{1}{\cancel{2}}\underset{1}{\cancel{x}})} = \frac{x\sqrt{2x}}{3}$$

(The smallest multiple of $2x$ which is a perfect square is $4x^2$.)

Simplify these radical expressions by rationalizing the denominators.

1. $\sqrt{\frac{2}{3}} =$

2. $\sqrt{\frac{3}{5}} =$

3. $\sqrt[3]{\frac{2}{9}} =$

4. $\sqrt[3]{\frac{3}{25}} =$

5. $\sqrt{\frac{1}{8}} =$

6. $\sqrt{\frac{2}{7}} =$